ADAPTIVE TESTING IN THE TWO SAMPLE SCALE PROBLEM

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I. INTRODUCTION

An ice cream factory is considering two different brands of dispensers to fill their cartons. Both brands can be adjusted to the desired number of ounces, and this amount is automatically dispensed at regular intervals. The company is concerned that Brand S (which is considerably less expensive than Brand G) will not be as precise as Brand G in the amount of ice cream it puts into the cartons. Thus they are interested in testing the variability of the two brands of dispensers, and if Brand S is not significantly less precise in the amounts it is dispensing, they will use the less expensive brand. A more formal statement of their problem follows.

Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be independent random samples from continuous c.d.f.'s $F(x)$ and $G(y)$, respectively. Assume these distributions are identical except for scale. Let θ_x be the scale parameter of $F(x)$ and θ_y be the scale parameter of $G(y)$, and let $\theta = \theta_x/\theta_y$. The problem we consider in this report is the one tailed test H_0: $\theta=1$ vs. H_1: $\theta>1$.

The usual statistic for this test is $F = (S_x/S_y)^2$, where S is the sample standard deviation. We reject the null hypothesis if $F > F(\alpha, n-1, m-1)$. However, the F test supposes $F(x)$ and $G(y)$ to be normal c.d.f.'s and is known to be very sensitive to departures from this assumption. For example, Box (1953) discusses the problem and cites several previous references. Wasserstein (1987) shows through

simulation that under distributions other than the normal, the F test does not even retain the α level when testing at the null hypothesis. He discusses several alternative tests and compares their performance under various conditions. He further suggests the use of permutation tests based on functions of robust estimators such as trimmed means. In this study we will investigate the performance of such tests for the two sample scale problem presented above.

II. The Problem of Interest

A. Trimmed Means

Let $x_1 < \ldots < x_n$ be an ordered sample of size n from a population with distribution function F(x). The α percent trimmed mean is defined (Boyer and Kolson (1983)) by

$$m(\alpha) = \frac{1}{n(1-2\alpha)} \left\{ \sum_{i=[n\alpha]+2}^{n-[n\alpha]-1} x_i + (1+[n\alpha]-n\alpha)(x_{[n\alpha]+1}+x_{n-[n\alpha]}) \right\}$$

Hence $m(\alpha)$ is the average of the sample values that remain after a proportion α have been "trimmed" from each end of the sample. The average of those discarded observations (i.e. the "mean of the trimmings") is defined:

$$m^c(\alpha) = \frac{1}{2n\alpha} \left\{ \sum_{i=1}^{[n\alpha]} (x_i + x_{n-i+1}) + (n\alpha-[n\alpha])(x_{[n\alpha]+1}+x_{n-[n\alpha]}) \right\}$$

We note that commonly used estimators can be thought of as limiting forms of trimmed means: $m(.5)$ and $m^c(0)$ are defined respectively to be the median and midrange, while $m(0) = m^c(.5)$ is the mean. Each of these three are the most efficient estimators of location (in fact, they are UMVUE's) for different distributions, namely the midrange for the uniform distribution, the mean for the normal, and the median for the double exponential.

As an example, let $z = (1, 2, 3, 5, 9)$ be the sample vector. The twenty percent trimmed mean, $m(.2)$, is the average of the observations that remain after trimming $(.2)*(5)=1$ observation from each end of the sample, so $m(.2) = (2+3+5)/3 = 10/3$. The average of those two trimmed observations is $m^c(.2) = (1+9)/2 = 5$. According to the definition of $m^c(0)$ (the midrange) and because our sample size is five, $m^c(0) = m^c(.2) = 5$. The median is $m(.5) = 3$, and the mean of this sample is $m(0) = (1+2+3+5+9)/5 = 4$.

Note that the definition allows for fractional parts of observations to be used if $n\alpha$ is not an integer. For example, $m^c(.3)$ is the average of the smallest 1.5 observations (i.e. 1 and .5*2) and the largest 1.5 observations (i.e. 9 and .5*5) so $m^c(.3) = (1 + 1 + 9 + 2.5) / 2*5*.3 = 13.5/3 = 4.5$.

B. Test Statistics

Since, as previously noted, trimmed means efficiently estimate location in various distributions, we speculate that functions of these trimmed means might be efficent estimators of scale. Thus in this study, we estimate the scale parameter of both populations, then use a test statistic which is the ratio of those two estimates, as in the F-test. The scale estimators can be defined as follows: Let $m(\alpha)$ denote the α percent trimmed mean of a sample $z_1, \ldots, z_n$. Subtract $m(\alpha)$ from each sample value and square those deviations, yielding $w_1, \ldots, w_n$, say. Then find the same α percent trimmed mean of the w_i's. The square root of this trimmed mean is our estimator of scale. The definition follows similarly for $m^c(\alpha)$, the α percent mean of trimmings. It is readily seen that these estimators are invariant to changes in location, so that we need not even assume our populations are identical in location.

To illustrate our method of estimating scale, again let the sample vector be $z = (1, 2, 3, 5, 9)$. We will calculate estimates of scale based on all five trimmed means that were demonstrated in the previous section. We determined that $m^c(0) = m^c(.2) = 5$. Let v be the vector of deviations from 5, then $v = (-4, -3, -2, 0, 4)$, and the vector of squared deviations is $w = (0, 4, 9, 16, 16)$. The twenty percent mean of trimmings for w is $(0+16)/2 = 8$, so the estimate of scale based on $m^c(.2)$ (and $m^c(0)$) is $\sqrt{8} = 2.83$.

For $m(0) = 4$, $w = (1, 1, 4, 9, 25)$ and the estimate of the scale parameter has value $\sqrt{(1+1+4+9+25)/5} = \sqrt{8} = 2.83$. Since $m(.5) = 3$, the median based scale estimate is $\sqrt{4} = 2$, as computed from $w = (0, 1, 4, 4, 36)$. Finally, $m(.2) = 10/3$, so $v = (-7/3, -4/3, -1/3, 5/3, 17/3)$, and $w = (1/9, 16/9, 25/9, 49/9, 289/9)$. The twenty percent trimmed mean of w is $[(16+25+49)/9]/3 = 10/3$, and scale is estimated as $\sqrt{10/3} = 1.83$.

If we use $m(0)$ (i.e. the mean) as the basis for estimating scale, then our estimator is the square root of $\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$, which is the usual estimator of variance (using n rather than n-1). Hence our test statistic is the square root of the F test statistic. Using $m(.5)$, scale is estimated as the median deviation from the median, another common estimator, and the midrange type estimator is very nearly the range estimator of scale. Thus, certain of the tests examined in this report closely correspond to statistics currently in use.

The estimators of scale employed here may not be (in fact, they probably are not) unbiased estimators of θ_x or θ_y. However, $\hat{\theta}_x$ is an unbiased estimator of $c\theta_x$, for some constant c, and $\hat{\theta}_y$ is similary an unbiased estimator of $c\theta_y$. Hence the ratio $\hat{\theta}_x/\hat{\theta}_y$ is a reasonable estimate of $c\theta_x/c\theta_y = \theta_x/\theta_y = \theta$.

C. A family of symmetric distributions

Prescott (1978) discusses the robustness properties of trimmed means and means of trimmings as unbiased estimators of the location parameter μ in the exponential power family of distributions defined (Hogg (1972)) by the density function

$$f(x) = \{2\,\Gamma(1 + 1/\tau)\}^{-1}\, e^{-|x-\mu|^{\tau}} \qquad \{-\infty < x < \infty\ ,\ \tau \geq 1\}$$

The distributions in this family are symmetric about μ with variance $\Gamma(3/\tau)/\Gamma(1/\tau)$. If we let $\gamma = 1/\tau$ be a continuous parameter in the interval [0,1], this family can be shown to contain distributions which range from the uniform (γ=0) through short-tailed symmetric distributions to the normal (γ=1/2), then through long-tailed symmetric distributions to the double exponential (γ=1). This family of distributions will be referred to throughout the remainder of this report as the Prescott family.

D. Adaptive Estimation and Testing

Prescott (1978) also discusses the use of an adaptive scheme for estimating location in this family. Several adaptive statistics are proposed whereby the trimming proportion α is based upon a measure of nonnormality or tailweight. In particular, Prescott (1978) and Boyer and Kolson (1983) have shown the following to be the preferred estimator for small sample sizes ($n<50$) such as are used in this study.

$$T = \begin{cases} m^c(0.2) & \hat{Q} < 2.2 \\ m^c(0.3) & 2.2 \le \hat{Q} < 2.4 \\ m(0) & 2.4 \le \hat{Q} \le 2.8 \\ m(0.2) & 2.8 < \hat{Q} \le 3.0 \\ m(0.3) & 3.0 < \hat{Q} \end{cases}$$

The choice of location estimator for this statistic is based on a measure of nonnormality proposed by Hogg (1974), namely

$$\hat{Q} = (\bar{U}_{(0.05)} - \bar{L}_{(0.05)}) \,/\, (\bar{U}_{(0.5)} - \bar{L}_{(0.5)}),$$

where $\bar{U}_{(\beta)}$ and $\bar{L}_{(\beta)}$ are the average of the largest and smallest $n\beta$ order statistics, respectively, with fractional items used if $n\beta$ is not an integer. The choice of $\hat{Q}$ over other measures of tailweight such as kurtosis is discussed in detail by Hogg (1972, 1974) and Prescott (1978), as well as the choice of the 5 and 50% proportions.

We use T as the basis for an adaptive procedure in testing for equality of scale. The failure of the F test in non-normal distributions motivates the use of an adaptive procedure. We first estimate non-normality using $\hat{Q}$, then select a scale estimator based on the trimmed means specified in T. If $\hat{Q}$ suggests the distribution is normal, we estimate scale based on the mean, which is equivalent to using the Permutation F Test to test our hypothesis. Otherwise, we use a trimmed mean or mean of trimmings as the basis for estimating scale.

In this problem we have two samples but wish to use the same scale estimator, i.e. the same trimming proportion, for both samples. Since Q is invariant to changes in scale, for each particular distribution $Q_x = Q_y$, so $\hat{Q}_x$ should be approximately equal in value to $\hat{Q}_y$. To avoid the possibility of slight variations in the two estimates causing selection of different trimming proportions, we let $\hat{Q} = \frac{1}{2}(\hat{Q}_x + \hat{Q}_y)$ and use T to determine the amount of trimming to be used in both samples. We then estimate scale and form our test statistic in the manner that was described in section B of chapter II.

E. Permutation Tests

Since the distribution of the test statistics used in this study are not mathematically tractable, we use a randomization procedure to perform the test of hypothesis. Dwass (1957) gives a more rigorous definition of permutation tests than will be presented here. Our purpose is to explain the procedure in this context.

Suppose $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ are two independent random samples from continuous distributions, with

$$z = (z_1, \ldots, z_n, z_{n+1}, \ldots, z_{n+m}) = (x_1, \ldots, x_n, y_1, \ldots, y_m)$$

being the combined sample of size $N = n+m$. Let $u(z)$ be a statistic based on z and let $t=u(z)$ be the value of $u(\cdot)$ for the observed z.

Consider the $r = \frac{N!}{n!m!}$ permutations of the indices of z which divide z into two subsamples. The set $u_1,\ldots,u_r$ comprises the permutation sampling distribution of the statistic $u(\cdot)$. Note we make no distributional assumptions about $u(\cdot)$. Now compare t to this sampling distribution. If k of the u_i are as extreme or more extreme than t, then the observed p-value for this test is k/r.

If indeed the null hypothesis of no scale differences is true, then the populations are identical. In that circumstance, we can think of randomly assigning the labels X and Y to the observations, or equivalently, randomly dividing z into two subsets. The observed statistic t is thus, under H_0, a randomly chosen element from the distribution of $u(\cdot)$, the set of all possible such elements. On the average, t will have a value at or near the mean of $u(\cdot)$, and such a value is unlikely to lead to a conclusion in favor of an alternative hypothesis. It is important to note that this test is conditional upon the data itself. However, the permutation test procedure does have an overall significance level α (Randles and Wolfe (1979)) regardless of the underlying distribution.

While the permutation test is intuitively appealing, there is one inherent problem. For small sample sizes, the permutation set is relatively short and easily enumerable. For example, if n=m=3, there are only 20 possible permutations. However, for n=m=10, there are 184,756 possible permutations to consider, too large a set to evaluate in practice (especially in a study involving runs of 1000

replications each!). Thus, a subset sampling approach first suggested by Dwass (1957) holds considerable merit. We randomly sample 500 out of the set of all permutations, and calculate u(z) for each of those 500. If 20 of the u(z) are more extreme than t, our p-value is 20/500 = 0.04, which is an estimate of the actual significance level we would have observed by evaluating all 184,756 permutations.

To determine if 500 sampled permutations is sufficient to estimate the actual significance level of the test, we examined the power of four of our tests for one distribution (the double exponential) at six sizes of permutation subset sampling. We were looking for stability in the power estimates; if 500 samples gave approximately the same estimate of power as 1500 samples, then there would not be a need to use 1500.

Wasserstein (1987) showed that a test based on 1600 samples is highly comparable to full enumeration for this same problem. We looked at subsets of 100, 250, 500, 750, 1000 and 1500 permutations. At the null hypothesis (i.e. $\theta = \theta_y/\theta_x = 1$) there is virtually no difference in either the .01 or .05 rejection rates across the different sizes of subsets. (See Table II.E, which is based on 500 replications of the simulation.) At θ=2 and θ=4, there is a substantial power difference between a subset of 100 and the other subsets, but once the subset size is increased to 250, the rejection rates stabilize. Thus we do not seem to gain substantial accuracy by choosing subsets of 1500 or even 1000 over subsets of 500.

TABLE II.E Comparison of Power at Different Levels of Subsampling
.01 Rejection Rates
.05 Rejection Rates

	$m^c(0)$			$m^c(.5)$		
	$\theta=1$	$\theta=2$	$\theta=4$	$\theta=1$	$\theta=2$	$\theta=4$
100	.010 .040	.124 .326	.444 .764	.012 .042	.118 .352	.480 .806
250	.010 .040	.142 .346	.528 .790	.010 .040	.140 .360	.528 .846
500	.008 .040	.134 .344	.506 .784	.010 .044	.118 .364	.556 .844
750	.010 .042	.144 .348	.530 .786	.010 .044	.138 .376	.564 .846
1000	.008 .044	.132 .344	.514 .780	.010 .044	.126 .366	.566 .840
1500	.010 .044	.140 .344	.502 .782	.010 .044	.120 .374	.558 .836

	m(.5)			adaptive		
	$\theta=1$	$\theta=2$	$\theta=4$	$\theta=1$	$\theta=2$	$\theta=4$
100	.004 .058	.058 .252	.302 .670	.014 .052	.116 .354	.484 .820
250	.010 .054	.086 .264	.388 .690	.010 .044	.158 .376	.594 .860
500	.010 .044	.064 .260	.354 .684	.012 .048	.140 .368	.562 .852
750	.008 .052	.078 .268	.374 .690	.012 .046	.150 .378	.572 .850
1000	.008 .052	.068 .264	.362 .696	.010 .048	.142 .368	.566 .846
1500	.008 .054	.076 .253	.348 .698	.010 .050	.142 .374	.566 .848

III. A Simulation Study

A. Scope of the Simulation

We compare by simulation the power of eight randomization tests, each based on robust estimators of scale. These eight tests will be referred to according to the trimmed mean or mean of trimmings used in estimating the scale parameter. One of these tests uses the adaptive estimation statistic T described in section D of chapter II. The other seven use fixed levels of α (the trimming proportion). Five of these comprise the adaptive statistic; the median and midrange are also used. Hence the eight statistics are based on functions of the following trimmed means:

1) $m^c(0)$ -- the midrange
2) $m^c(0.2)$
3) $m^c(0.3)$
4) $m^c(0.5) = m(0)$ -- the mean
5) $m(0.2)$
6) $m(0.3)$
7) $m(0.5)$ -- the median
8) the adaptive statistic, which uses one of 2) through 6) based on the observed value of the statistic $\hat{Q}$.

The tests were compared under several symmetric distributions, with sample sizes of 10 and 10. Five values of γ were chosen to

represent the exponential power family of distributions defined in section II.C : $\gamma=0$ (the uniform distribution); $\gamma=0.25$; $\gamma=0.5$ (the normal); $\gamma=0.75$; and $\gamma=1.0$ (the double exponential). We also used the Cauchy and 10% Mixed Normal, which consists of 90% N(0,1) contaminated with 10% N(0,64). These two distributions were used by Wasserstein (1987), and we also used them because his work on the same problem prompted this study. In addition, these distributions tend to have heavier tails than any of the members of the Prescott family.

Let μ_x and θ_x be, respectively, the location and scale parameters of population 1, and let μ_y and θ_y be the location and scale parameters of population 2. In the simulation, $\mu_x = \mu_y = 0$, which causes no loss of generality since all the tests are location invariant. Let $\theta = \theta_y / \theta_x$. Four values of θ are considered in each distribution to provide a wide range of power estimates. The results appear in Appendix 2.

B. Description of the Simulation Program

This simulation was actually executed in two parts. Part one consisted of generating the sample values through IMSL subroutines on an NAS 6630 (National Advanced System) mainframe. The remainder of the simulation was also written in Fortran but implemented on a Harris 700 computer. Both programs are listed in Appendix 1.

The required input for the sample generation program is as follows: number of replications, sample sizes (n,m), the value of γ (To generate from the Cauchy, set γ=1.25, for the Mixed Normal, set γ=1.50. This is for convenience only, and is not meant to imply that these distributions belong to the Prescott family.), the values of θ_x and θ_y and the seeds for the random number generators. These values and the sample data are then output to a file which is used as input for the second part of the simulation. The Prescott family can be derived via a power transformation from the gamma distribution with scale parameter 1 and shape parameter γ, and this method was used to generate these distributions.

The simulation program consists of four main parts, which are discussed here in some detail.

1) Input all parameters associated with sample generation, along with a seed for the random number generator in the permutation test. Set all arrays to zero.

2) Input the two samples, which are then combined and sorted (for use in the permutation test). Calculate each of the test statistics based on the original data. For the adaptive statistic, only $\hat{Q}$ and the interval in which $\hat{Q}$ falls is calculated, since T will always use one of the statistics previously calculated.

3) Run the approximate permutation test by sampling 500 out of the entire set of permutations, without replacement. Calculate each test statistic and compare the permutation value to the original value for each statistic. Calculate an approximate p-value as e/500, where e is the number of permutation statistic values more extreme than the original. To mimimize the run time of the simulation, whenever e exceeds 25 (5% of 500) for a particular statistic, discontinue calculation of that statistic. If e is greater than 25 for all statistics, then exit the permutation test.

The 500 permutation samples are generated in the following way. Let $N=n+m$. A set of n random integers between 1 and N are randomly selected without replacement, representing the indices of the items in the combined sample to be assigned to the first sample, with the remaining items assigned to the second sample. The statistics are then calculated from these two samples.

4) Note which tests are significant at the $\alpha=.05$ level. Repeat steps 2 and 3 as desired (1000 times in this study). Calculate .05 rejection rates, the average number of permutations sampled and the mean and variance of $\hat{Q}$.

Figure III.B gives a partial list of the subroutines used in the simulation program.

FIGURE III.B List of Subroutines Used in the Simulation

BPERM	Executes the permutation test
DEVSQ	Calculates two vectors of squared deviations around corresponding location estimates
MEAN MEDIAN MIDRAN	Calculate the sample mean, median and midrange, for each of two samples.
QHAT	Calculates an estimate of Q, Hogg's nonnormality indicator
QINT	Determines the interval in which Q is observed by which α (the trimming proportion) is adaptively chosen
SAMPER	Chooses the permutation sample from the set of all possible permutations
SHELL	Performs a shell sort
TCMEAN	Calculates the α mean of trimmings
TMEAN	Calculates the α timmed mean
TMNSCL	Calculates estimates of scale based on the trimmed mean (similar for TCMNSC, MNSCAL, MEDSCL, and MIDSCL)

C. Results of the Simulation Study

The simulation results are presented in three sections. In the first, we compare the power of the eight tests under the various distributions. The second section examines the performance of $\hat{Q}$ as

an estimator of Q. In the third section, we discuss a time saving method of performing the permutation test.

1. Power Comparisons

The reader should refer to Tables A-1 through A-8 and Figures B-1 through B-8 in Appendix 2. The findings can be summarized as follows.

1) The means of trimmings ($m^c(0)$, $m^c(.2)$, $m^c(.3)$) perform better than either the 20% or 30% trimmed means for the short- to medium- tailed distributions, but the opposite is true for the long tailed Cauchy and 10% Mixed Normal, where the trimmed means perform far better. In fact, for the 10% Mixed Normal, the tests based on the 20% and 30% trimmed means are the most powerful tests. They outperform any of the "standard" tests (those based on the midrange, mean and median) and the adaptive test. This was the only distribution where one of those four was not the most powerful.

2) The mean test performs well for all except the Cauchy and Mixed Normal, but even for those distributions its power is greater than the other means of trimmings. Also the test performs better than might be expected for the Double Exponential.

3) The median test did not perform well at all except for the Cauchy and Mixed Normal; even there it was not the most powerful

test. The median test does not perform well even for the Double Exponential, where we might expect that it would.

4) The adaptive estimation test consistently performs well, especially for the heavy-tailed distributions. It is always in the top group of tests in terms of power. No other statistic is so consistent.

Thus while the adaptive statistic does not always yield the single most powerful test, under no distribution is any other test clearly more powerful than the adaptive. In fact, no test is the overwhelming favorite for any distribution.

2. Performance of $\hat{Q}$

We calculated average values of $\hat{Q}$ (with standard errors) for each run of the simulation. These results are presented for the four values of θ examined in each distribution, along with the true population value of Q. As can be seen in Table III.C below, the statistic $\hat{Q}$ is invariant to changes in scale, but, as noted by Boyer and Kolson (1983), tends to underestimate the population value Q. For the Uniform distribution, this error is not substantial ($\hat{Q}$ averages 1.85 when $Q = 1.90$) but as the tailweight of the population increases, the degree of under-estimation becomes more severe.

TABLE III.C Observed Values of $\hat{Q}$ Compared with Population Values

Average Values of $\hat{Q}$
Standard error of estimate

	Q	θ_1	θ_2	θ_3	θ_4
Uniform	1.90	1.842 .213	1.851 .210	1.848 .207	1.847 .203
Prescott(.25)	2.20	1.952 .226	1.963 .231	1.969 .224	1.955 .221
Normal	2.58	2.109 .265	2.096 .258	2.111 .260	2.116 .265
Prescott(.75)	2.95	2.240 .290	2.262 .298	2.266 .282	2.251 .286
Double Exp	3.30	2.363 .323	2.392 .310	2.371 .308	2.392 .330
Mixed Normal	4.95	2.677 .521	2.656 .521	2.680 .510	2.690 .503
Cauchy	10.00	3.095 .579	3.102 .594	3.114 .596	3.132 .594

For example, in the case of the Double Exponential, the average $\hat{Q}$ is 2.38 for a population value of Q = 3.30; Q = 10.0 for the Cauchy but the average $\hat{Q}$ is 3.11. At the completion of this project we discovered that when n=10 the numerator of $\hat{Q}$ actually estimates the upper and lower 10% rather than 5% of the distribution, so that the population values of Q for this special case are smaller than the general values which appear in the table above. For example, at

n=10 the population values of Q are 5 for the Cauchy, and 3.4 for the 10% Mixed Normal. Hence the values of Q which we observed do not show such marked underestimation. The fact that our adaptive procedure displayed such consistently high power even under these conditions suggests that only crude estimates of tailweight are necessary for this test to perform well.

3. A Permutation Test Short-Cut

In thia aimulation, we were only interested in .05 rejection rates. Thus, for any given replication, if rejection at the .05 level became impossible (because more than 25 of the permutation values were more extreme than the original value) the test was terminated. For runs of the simulation at the null hypothesis (i.e. $\theta = \theta_y/\theta_x = 1$) an average of only 150 (approximately) sampled permutations were necessary. For the cases of the most extreme departures from the null hypothesis which we examined, an average of 493 permutation were required. This disparity resulted in a ratio of almost 5 to 1 in CPU minutes required to complete the simulation (a maximum of 635 CPU minutes to a minimum of 130), a substantial time savings. Thus in an actual application of the permutation test, one might wish to consider 500 to 1000 samples of the set of permutations, but only continue evaluation of the statistic u(•) while H_0 can still be rejected at the desired level of α.

IV. Conclusion

We have seen that, in general, randomization tests based on functions of trimmed means perform well for the two sample scale problem. In particular, the test based on the mean (which is the permutation F test) is quite powerful for all except the heaviest tailed distributions. The adaptive test is by far the most consistent of the tests we have examined here. Based on this finding we recommend the use of the adaptive test for this problem. We also recommend the permutation test shortcut discussed in section III.C.3. Continued research in this area could examine the power of this adaptive procedure for sample sizes other than 10 and 10, and consideration of the problems posed by unequal sample sizes. We believe the adaptive statistic will continue to display the desirability it has shown here.

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APPENDIX 1
Source Listing of Simulation Program

```
C  GENERATION PROGRAM
C    PURPOSE
C      GENERATES THE SAMPLES FROM VARIOUS DISTRIBUTIONS
C      FOR THE SIMULATION
C
C
C  DEFINE VARIABLE NAMES
C
C  ID             -  INDICATES SAMPLINC DISTRIBUTION
C  SAMPL 1,2      -  REAL*8 ARRAY OF SAMPLE VALUES FROM POP'N 1,2
C  N,M            -  SAMPLE SIZES
C  NREPS          -  NUMBER OF INDEPENDENT REPLICATIONS DESIRED
C  GAMMA          -  PARAMETER OF THE PRESCOTT FAMILY
C  THETA 1,2      -  ACTUAL SCALE PARAMETERS OF POPULATION 1,2
C  MT 1,2         -  ADDITIONAL SCALE PARMS FOR MIXED NORMAL DIST'N
C  IX,JX,KX,LX -  SEEDS FOR THE RANDOM NUMBER GENERATORS
C  DIX,DJX,DKX,DLX  -  DOUBLE PRECISION VAR'S WITH SEEDS VALUES FOR RNC
C
      PROCRAM CEN
      REAL*8 SAMPL1(10),SAMPL2(10),R,A,B,T,DIX,DJX,DKX,DLX,PI
C
      REAL*4 CAMMA,X(10),Y(10),WK(50),BETA1,BETA2,THETA1,THETA2,MT1,MT2
C
      INTECER*4 NREPS,IX,JX,KX,LX,N,M,ID
C
      CHARACTER*15 IDENT
C
      COMMON/RNC/DIX,DJX,DKX,DLX
C
      DATA NREPS,N,M,CAMMA/1000,10,10,0.00/
      DATA THETA1,THETA2,MT1,MT2/1.,1.,0.,0./
C
      READ(5,240) IX,JX,KX,LX
      WRITE(6,240) IX,JX,KX,LX
        DIX=IX
        DJX=JX
        DKX=KX
        DLX=LX
C
C   CENERATE THE SAMPLES
C
      DO 170  J=1,NREPS
      ID = 4.*CAMMA + 1
      COTO (100,110,110,110,110,120,130),ID
```

```
C
  100 CALL UNIFOR(SAMPL1,SAMPL2,N,M,THETA1,THETA2)
      IDENT = 'UNIFORM'
      COTO 150
C
  110 CALL PRESCT(SAMPL1,SAMPL2,N,M,THETA1,THETA2,GAMMA)
       GOTO (111,112,113,114,115),ID
  111  COTO 150
  112   IDENT = 'PRESCOTT(.25)'
        COTO 150
  113   IDENT = 'NORMAL'
        COTO 150
  114   IDENT = 'PRESCOTT(.75)'
        COTO 150
  115   IDENT = 'DOUBLE EXPON'
        COTO 150
C
  120 CALL CAUCHY(SAMPL1,SAMPL2,N,M,THETA1,THETA2)
       IDENT = 'CAUCHY'
       COTO 150
C
  130 CALL MIXED(SAMPL1,SAMPL2,N,M,THETA1,THETA2,MT1,MT2)
       IF (J .GT. 1)  GOTO 140
      IDENT = 'MIXED NORM'
      WRITE(6,200) IDENT,NREPS
      WRITE(6,220) N,M,THETA1,THETA2,MT1,MT2
  140 WRITE(6,230) (SAMPL1(I),I=1,N)
      WRITE(6,230) (SAMPL2(I),I=1,M)
      GOTO 170
C
  150  IF (J .GT. 1)  GOTO 160
      WRITE(6,200) IDENT,NREPS
      WRITE(6,210) N,M,THETA1,THETA2
  160 WRITE(6,230) (SAMPL1(I),I=1,N)
      WRITE(6,230) (SAMPL2(I),I=1,M)
C
  170 CONTINUE
C
      STOP
C
C      DEFINE OUTPUT FORMATS
C
  200 FORMAT(1X,A15,I5)
  210 FORMAT(2I5,2F10.5)
  220 FORMAT(2I5,4F10.5)
  230 FORMAT(10F8.4)
  240 FORMAT(4I10)
C
      END
C
C
```

```
G***********************************************************************
G
G  SUBROUTINE GAUCHY
C    PURPOSE
C      GENERATES TWO SAMPLES OF SIZES N AND M, RESPECTIVELY, FROM
C      THE GAUGHY DISTRIBUTION WITH LOCATION PARAMETER ZERO AND SGALE
G      PARAMETERS BETA1 AND BETA2, RESPEGTIVELY.  USES THE PROBABILITY
G      INTEGRAL TRANSFORM TEGHNIQUE TO GENERATE CAUGHY DEVIATES FROM
C      UNIFORM DEVIATES.
C
G    USAGE
C      CALL GAUCHY(SAMPL1,SAMPL2,N,M,BETA1,BETA2)
G
C    SUBROUTINES CALLED
C      GGUBFS
G
G    DESGRIPTION OF PARAMETERS
C      SAMPL1  -  REAL*8 ARRAY OF LENGTH N GONTAINING THE SAMPLE VALUES
G                 FROM POPN 1
C      SAMPL2  -  REAL*8 ARRAY OF LENGTH M GONTAINING THE SAMPLE VALUES
C                 FROM POPN 2
C      N,M     -  SAMPLE SIZES
C      BETA1   -  SCALE PARAMETER OF POPN 1
C      BETA2   -  SCALE PARAMETER OF POPN 2
C
C
      SUBROUTINE GAUGHY(SAMPL1,SAMPL2,N,M,BETA1,BETA2)
      INTEGER*4 N,M
      REAL*8 SAMPL1(N),SAMPL2(M),DIX,DJX,DKX,DLX,PI
      REAL*4 BETA1,BETA2,A,B
      COMMON/RNG/DIX,DJX,DKX,DLX
      DATA PI/3.141592654/
C
      DO 100  I=1,N
      A = GGUBFS(DIX)
  100 SAMPL1(I) = BETA1 * TAN(PI*(A-.5))
C
      DO 110  I=1,M
      B = GGUBFS(DJX)
  110 SAMPL2(I) = BETA2 * TAN(PI*(B-.5))
C
G
      RETURN
      END
G
G
```

```
C************************************************************************
C  SUBROUTINE MIXED
C    PURPOSE
C      GENERATES TWO SAMPLES OF SIZES N AND M, RESPECTIVELY, FROM
C      A 10% MIXED NORMAL WITH SCALE PARAMETERS THETA1 AND THETA2
C      FOR 90% OF THE SAMPLE, AND MIXING SCALE PARAMETERS MT1 AND
C      MT2 FOR THE REMAINING 10 %.  (THE SCALE PARAMETERS ARE
C      STANDARD DEVIATIONS)
C
C    USAGE
C      CALL MIXED(SAMPL1,SAMPL2,N,M,THETA1,THETA2,MT1,MT2)
C
C    SUBROUTINES/FUNCTIONS CALLED
C      GGNPM, GGUBFS
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1,2 -  REAL*8 ARRAY OF LENGTH N CONTAINING THE SAMPLE VALUES
C                  FROM POP'N 1,2
C      N,M      -  SAMPLE SIZES
C      THETA1,2 -  STANDARD DEVIATION OF POPN 1,2
C      MT1,2    -  STANDARD DEVIATION OF THE MIXING POPULATIONS
C
C    METHOD
C      CALLS SUBROUTINE GGNPM TO OBTAIN THE N(0,1) RANDOM DEVIATES,
C      THEN ADJUSTS THEM TO HAVE CORRECT VARIANCE
C
      SUBROUTINE MIXED(SAMPL1,SAMPL2,N,M,THETA1,THETA2,MT1,MT2)
      REAL*8 SAMPL1(N),SAMPL2(M),DIX,DJX,DKX,DLX
      REAL*4 X(10),Y(10),THETA1,THETA2,MT1,MT2,T,R
      INTEGER*4 N,M
      COMMON/RNG/DIX,DJX,DKX,DLX
C
        CALL GGNPM(DIX,N,X)
C
        DO 100  I=1,N
        T=THETA1
        R=GGUBFS(DKX)
        IF(R .LT. .10)T=MT1
        SAMPL1(I)=X(I)*T
100   CONTINUE
C
        CALL GGNPM(DJX,M,Y)
C
      DO 110 I=1,M
        T=THETA2
        R=GGUBFS(DLX)
        IF(R .LT. .10)T=MT2
        SAMPL2(I)=Y(I)*T
110   CONTINUE
C
      RETURN
      END
```

```
C*****************************************************************
C
C  SUBROUTINE PRESCT
C
C    PURPOSE
C      GENERATES TWO SAMPLES OF SIZE N AND M, RESPECTIVELY, FROM
C      THE PRESCOTT FAMILY OF SYMMETRIC DISTRIBUTIONS DEFINED BY
C      GAMMA IN THE INTERVAL (0,1), HAVING SCALE PARAMETERS BETA1
C      AND BETA 2
C
C    USAGE
C      CALL PRESCT(SAMPL1,SAMPL2,N,M,THETA1,THETA2,GAMMA)
C
C    SUBROUTINES CALLED
C      GGAMR, GGUBFS
C
C    DESCRIPTION OF PARAMTERS
C      SAMPL1  -  REAL*8 ARRAY OF LENGTH N CONTAINING THE SAMPLE
C                 VALUES FROM POPULATION 1
C      SAMPL2  -  REAL*8 ARRAY OF LENGTH M CONTAINING THE SAMPLE
C                 VALUES FROM POPULATION 2
C      BETA1,2 -  SCALE PARAMETER OF POPULATION 1,2
C      GAMMA   -  PRESCOTT FAMILY PARAMETER
C
C    METHOD
C      CALL SUBROUTINE GGAMR TO OBTAIN GAMMA(GAMMA,1) DEVIATES,
C      MAKES A POWER TRANSFORMATION TO THE APPOPRIATE PRESCOTT
C      DISTRIBUTION, AND ADJUSTS TO THE CORRECT SCALE
C
C
       SUBROUTINE PRESCT(SAMPL1,SAMPL2,N,M,BETA1,BETA2,GAMMA)
       REAL*8 SAMPL1(N),SAMPL2(M),R,DIX,DJX,DKX,DLX
       REAL*4 GAMMA,X(10),Y(10),WK(20),BETA1,BETA2
       INTEGER*4 N,M
       COMMON/RNG/DIX,DJX,DKX,DLX
C
       CALL GGAMR(DIX,GAMMA,N,WK,X)
C
       DO 100  I=1,N
        SAMPL1(I) = (X(I) ** GAMMA) * BETA1
        R = GGUBFS(DKX)
  100   IF (R .LT. 0.5)  SAMPL1(I) = -1 * SAMPL1(I)
C
       CALL GGAMR(DJX,GAMMA,M,WK,Y)
C
       DO 110  I=1,M
        SAMPL2(I) = (Y(I) ** GAMMA) * BETA2
        R = GGUBFS(DLX)
  110   IF (R .LT. 0.5)  SAMPL2(I) = -1 * SAMPL2(I)
C
       RETURN
       END
```

```
C***********************************************************************
C
C  SUBROUTINE UNIFOR
C    PURPOSE
C      CENERATES TWO SAMPLES OF SIZES N AND M FROM U(-THETA1,THETA1)
C      AND U(-THETA2,THETA2), RESPECTIVELY.
C
C    USACE
C      CALL UNIFOR(SAMPL1,SAMPL2,N,M,THETA1,THETA2)
C
C    FUNCTION CALLED
C      GGUBFS
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1  -  REAL*8 ARRAY OF LENCTH N CONTAININC THE SAMPLE VALUES
C                 FROM POPN 1
C      SAMPL2  -  REAL*8 ARRAY OF LENCTH M CONTAININC THE SAMPLE VALUES
C                 FROM POPN 1
C      N,M     -  SAMPLE SIZES
C      THETA1  -  SCALE PARAMETER OF POPN 1
C      THETA2  -  SCALE PARAMETER OF POPN 2
C
C    METHOD
C      INVOKES THE PRIME UNIFORM RANDOM NUMBER CENERATOR
C
      SUBROUTINE UNIFOR(SAMPL1,SAMPL2,N,M,BETA1,BETA2)
      REAL*8 SAMPL1(10),SAMPL2(10),DIX,DJX,DKX,DLX
      REAL*4 BETA1,BETA2,A,B
      INTECER*4 N,M
      COMMON/RNC/DIX,DJX,DKX,DLX
C
      DO 100 I=1,N
 99     A=GGUBFS(DIX)
        IF(A .LT. 0.000000001)COTO 99
        SAMPL1(I)=(A-.5)*2.*BETA1
100   CONTINUE
C
      DO 110 I=1,M
101    B=GGUBFS(DJX)
        IF(B .LT. 0.000000001)GOTO 101
        SAMPL2(I)=(B-.5)*2.*BETA2
110   CONTINUE
      RETURN
      END
```

```
C
C  SIMULATION PROCRAM
C    PURPOSE
C      COMPARE SIMILAR MEASURES OF SCALE BASED ON TRIMMED MEANS
C      FOR THE PRESCOTT FAMILY OF SYMMETRIC DISTRIBUTIONS, CAUCHY,
C      AND MIXED NORMAL DISTRIBUTIONS
C
C  VARIABLE DEFINITIONS
C
C  SAMPL 1,2  -  ARRAY OF SAMPLE VALUES FROM POPULATION 1,2
C  PSAMP 1,2  -  ARRAY OF SAMPLE VALUES AS ASSICNED IN THE
C                PERMUTATION TEST
C  SQDEV 1,2  -  ARRAY OF SQUARED DEVIATIONS (SEE SUB. DEVSQ)
C  COMB       -  ARRAY OF COMBINED SAMPLE VALUES
C  OSTAT      -  VALUES OF THE TEST STATISTICS EVALUATED ON
C                THE ORIGINAL SAMPLE DATA
C  PSTAT      -  VALUES OF THE TEST STATISTICS EVALUATED ON
C                THE PERMUTED SAMPLE DATA
C  EXTREM     -  ACCUMULATOR W/IN PERM. TEST OF EXTREM OBS.
C  REJECT     -  COUNTS REPS WHICH YIELDED SICNIFICANT PERM. TESTS
C  REJPER     -  PERCENT REJECTIONS FOR EACH STATISTIC
C  CONTIN     -  DETERMINES CONTINUATION OF PERMUTATION LOOP FOR
C                INDIVIDUAL STATISTICS
C  ALL        -  DETERMINES POINT OF TERMINATION OF PERM. LOOP
C  ODD        -  NOTES EVEN OR ODD SAMPLE SIZE FOR SUB. MEDI
C  N,M        -  SAMPLE SIZES
C  NREPS      -  NUMBER OF INDEPENDENT REPLICATIONS DESIRED
C  NPERM      -  NUMBER OF PERMUTATIONS TO BE SAMPLED
C  NSTAT      -  NUMBER OF STATISTICS TO BE TESTED
C  CVAL       -  CRITICAL VALUE OF EXTREM OBS. AT P=.05
C  ALPHA      -  DESIRED AMOUNT OF TRIMMINC
C  LOC 1,2    -  LOCATION ESTIMATOR FOR SAMPLE 1,2
C  SCALE 1,2  -  SCALE ESTIMATOR FOR SAMPLE 1,2
C  Q          -  NONNORMALITY INDICATOR USED IN THE ADAPTIVE SCHEME
C  INT        -  INDICATES THE INTERVAL (2,6) IN WHICH Q IS OBSERVED
C  ICT        -  VECTOR COUNTINC THE TIMES Q WAS PLACED IN EACH INTERVAL
C  QSUM       -  SUMS THE VALUES OF Q ( FOR MEAN Q)
C  QSQ        -  SUMS THE SQUARED VALUES OF Q (FOR VARIANCE OF Q)
C  IP         -  PERMUTATION COUNTER (USED AS A CHECK)
C  PSUM       -  SUMS THE NUMBER OF PERMUTATIONS NECESSARY FOR EACH REP
C  PCT        -  THE NUMBER OF REPS THE PERM TEST ENDED EARLY
C  ISAM       -  INDICATOR ARRAY FOR DIVISION OF SAMPLE IN PERM TEST
C  SEED       -  SEED FOR RANDOM NUMBER CENERATOR
C  THETA 1,2  -  ACTUAL SCALE PARAMETER FOR POPULATION 1,2
C  MT 1,2     -  ADDITIONAL SCALE PARMS FOR MIXED NORMAL
```

```
C
      PROGRAM SCALESIM
      REAL*6 SAMPL1(10),SAMPL2(10),PSAMP1(10),PSAMP2(10),COMB(20),
     1        SQDEV1(10),SQDEV2(10),OSTAT(10),PSTAT(10),SAMP(10),
     2        REJECT(10),REJPER(10),W(10),Z(10),SAMP1(10),SAMP2(10),
     3        LOC1,LOC2,SCALE1,SCALE2,QVAL1,QVAL2,Q,QHAT,X(10),Y(10),
     4        THETA1,THETA2,MT1,MT2,TM1,TM2,SC1,SC2,U,C,T,DIV,
     5        MEAN1,MEAN2,MEDI1,MEDI2,MIDRA1,MIDRA2,RATIO,CVAL,ALPHA,
     6        SUM1,SUM2,HOLD1,HOLD2,TSUM,TCSUM,TMEAN,TCMEAN,SEED,
     7        QSUM,QSQ,REPS,PSUM,AVEPERM,AVEQ,VARQ
C
      INTEGER*3 IP,IC,JC,N,M,INT,NREPS,NSTAT,NPERM,ICOMB(20),ISEED,I1,S,
     1            INUM,IDEN,NSAM,ISAM(20),NSIZE,ISTART,I2,EXTREM(10),PCT,
     2            ICT(10),GS(10)
C
      LOGICAL*3 CONTIN(10),ALL,ODD
C
      CHARACTER*15 IDENT,LABEL(10)
C
      COMMON/PERMCOM/OSTAT,NSTAT,N,M,COMB,INT,NPERM,CVAL
C
C  DEFINE OUTPUT FORMATS  .
C
    1 FORMAT(I5)
    2 FORMAT(8I5)
    3 FORMAT(1X,A15,I5)
    4 FORMAT(' THIS RUN INVOLVED SAMPLING FROM THE ',A15,'DISTRIBUTION')
    5 FORMAT('         WITH ',I5,' REPLICATIONS',/)
    6 FORMAT(2I5,2F10.5)
    7 FORMAT(2I5,4F10.5)
    8 FORMAT(' SAMPLE SIZES WERE: ',I5,' AND ',I5)
    9 FORMAT(' SCALE PARAMETERS WERE: ',F7.4,' AND ',F7.4,/)
   10 FORMAT(' SCALE PARAMETERS FOR SAMPLE 1: ',F7.4,' AND ',F7.4)
   11 FORMAT('                AND FOR SAMPLE 2: ',F7.4,' AND ',F7.4,/)
   12 FORMAT(10F8.4)
   13 FORMAT(' THE VALUE OF ',A15,'STATISTIC FOR THE ORIGINAL SAMPLE:',
     1F10.5)
   14 FORMAT(/,' THE PERMUTATION TEST ON THE ',I5,
     1'TH REPLICATION WAS TERMINATED AFTER ',I5,'  PERMUTATIONS',/,/)
   15 FORMAT(' EXTREM(',I1,'): ',I5,2X,' REJECT(',I1,'): ',F5.2)
   16 FORMAT(' REJECTION RATE FOR THE TEST BASED ON ',A15,'IS  ',F7.5,/)
   17 FORMAT(' NPERM:',I5,'  CVAL:',F8.4,' SEED:',F8.2,' NSTAT:',I5)
   18 FORMAT(1X,I5,' TIMES THE ADAPTIVE STATISTIC USED ',A15,/)
   19 FORMAT(' AVERAGE NUMBER OF PERMUTATIONS: ',F7.2,/,
     1        ' THE PERMUTATION TEST ENDED EARLY ',I5,' TIMES',/)
   20 FORMAT(' AVERAGE VALUE OF Q: ',F7.4,'   WITH VARIANCE: ',F7.4,/)
C
C
C       DEFINE NUMBER OF PERMUTATIONS
C         AND  NUMBER OF STATISTICS TO BE COMPARED
C       AND SET SEED FOR RANDOM NUMBER GENERATOR
C
```

```
      NPERM=500
      CVAL = 0.05*NPERM
      NSTAT=8
      READ(17,1)ISEED
        SEED=FLOAT(ISEED)
      CALL RANUP(SEED)
      WRITE(16,17)NPERM,CVAL,SEED,NSTAT
C
C           INITIALIZE ARRAYS
C
      DO  50  K=1,10
       REJECT(K) = 0
       OSTAT(K) = 0.0
       PSTAT(K) = 0.0
       CONTIN(K) = .TRUE.
       REJPER(K) = 0
       SAMPL1(K) = 0.0
       PSAMP1(K) = 0.0
       SAMP1(K) = 0.0
       SAMP(K) = 0.0
       SQDEV1(K) = 0.0
       X(K) = 0.0
       Z(K) = 0.0
       SAMPL2(K) = 0.0
       PSAMP2(K) = 0.0
       SAMP2(K) = 0.0
       SQDEV2(K) = 0.0
       Y(K) = 0.0
       W(K) = 0.0
   50  ICT(K) = 0
C
      DO 60 K=1,20
      ICOMB(K) = 0
      COMB(K) = 0.0
   60 ISAM(K) = 0
C
      PCT = 0
      PSUM = 0.0
      QSUM = 0.0
      QSQ = 0.0
C
      LABEL(1) = 'THE MIDRANCE'
      LABEL(2) = 'MC(.2)'
      LABEL(3) = 'MC(.3)'
      LABEL(4) = 'THE MEAN'
      LABEL(5) = 'M(.2)'
      LABEL(6) = 'M(.3)'
      LABEL(7) = 'THE MEDIAN'
      LABEL(8) = 'ADAPTATION'
C
      READ(15,2)(CS(I),I=1,8)
      WRITE(16,2)(CS(I),I=1,8)
```

```
C
C*****************************************************************
C
C                    BEGIN REPLICATION LOOP
C
C*****************************************************************
C
C
C          INPUT THE SAMPLES
C
      READ(15,3) IDENT,NREPS
      WRITE(16,4) IDENT
      WRITE(16,5) NREPS
C
      DO 200  J=1,NREPS
      IF (IDENT .EQ. 'MIXED NORM') GOTO 110
C
        IF (J .GT. 1)  GOTO 105
      READ(15,6) N,M,THETA1,THETA2
      WRITE(16,8) N,M
      WRITE(16,9) THETA1,THETA2
  105 READ(15,12) (SAMPL1(I),I=1,N)
      READ(15,12) (SAMPL2(I),I=1,M)
C*    WRITE(16,12) (SAMPL1(I),I=1,N)
C*    WRITE(16,12) (SAMPL2(I),I=1,M)
      GOTO 120
C
  110   IF (J .GT. 1)  GOTO 115
      READ(15,7) N,M,THETA1,THETA2,MT1,MT2
      WRITE(16,8) N,M
      WRITE(16,10) THETA1,MT1
      WRITE(16,11) THETA2,MT2
  115 READ(15,12) (SAMPL1(I),I=1,N)
      READ(15,12) (SAMPL2(I),I=1,M)
C*    WRITE(16,12) (SAMPL1(I),I=1,N)
C*    WRITE(16,12) (SAMPL2(I),I=1,M)
C
C          COMBINE AND SORT THE SAMPLES
C
  120 DO 125  I=1,N
  125 COMB(I) = SAMPL1(I)
      DO 130  I=1,M
  130 COMB(I+N) = SAMPL2(I)
C
      CALL SHELL(COMB,N+M)
C
```

```
C             CALCULATE THE STATISTICS
C
C         K=1 : MC(0) -- MIDRANCE
C         K=2 : MC(.2)
C         K=3 : MC(.3)
C         K=4 : MC(.5) = M(0) -- MEAN
C         K=5 : M(.2)
C         K=6 : M(.3)
C         K=7 : M(.5) -- MEDIAN
C         K=8 : ADAPTIVELY CHOSEN TO BE ONE OF THE ABOVE
C
C
      DO 190  K=1,NSTAT
         COTO (135,140,145,150,155,160,165,170),K
C
  135 CALL MIDSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
      COTO 175
C
  140 ALPHA=.2
      COTO 148
C
  145 ALPHA=.3
C
  148 CALL TCMNSC(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2,ALPHA)
      COTO 175
C
  150 CALL MNSCAL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
      COTO 175
C
  155 ALPHA=.2
      COTO 162
C
  160 ALPHA=.3
C
  162 CALL TMNSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2,ALPHA)
      COTO 175
C
  165 CALL MEDSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
      COTO 175
C
  170 CALL QINT(SAMPL1,SAMPL2,N,M,Q,INT)
      OSTAT(K) = OSTAT(INT)
       ICT(INT) = ICT(INT) + 1
       QSUM = QSUM + Q
       QSQ = QSQ + Q ** 2
      COTO 190
C
  175 OSTAT(K) = RATIO(SCALE1,SCALE2)
C*180  WRITE(16,13)LABEL(K),OSTAT(K)
  190 CONTINUE
C
```

```
C
C          RUN THE PERMUTATION TEST
C
C
      CALL BPERM(ALL,EXTREM,IP)
C
        PSUM = PSUM + IP
        IF (IP .LT. NPERM) PCT = PCT + 1
C
        IF (ALL) GOTO 192
C
C*      WRITE(16,14)J,IP
        GOTO 200
C
  192   DO 195  K=1,NSTAT
        IF (EXTREM(K) .LT. CVAL) REJECT(K) = REJECT(K) + 1.0
C*      WRITE(16,15)K,EXTREM(K),K,REJECT(K)
  195   CONTINUE
C
  200 CONTINUE
C
C****************************************************************
C
C                END OF REPLICATION LOOP
C
C****************************************************************
C
C
C         CALCULATE SUMMARY STATISTICS
C
C
        REPS = FLOAT(NREPS)
      DO 210  K=1,NSTAT
         REJPER(K) = REJECT(K) / REPS
         WRITE(16,16) LABEL(K),REJPER(K)
  210 CONTINUE
       DO 220  K=2,6
  220  WRITE(16,18) ICT(K),LABEL(K)
C
      AVEPERM = PSUM /REPS
      AVEQ = QSUM / REPS
      VARQ = (QSQ - (QSUM**2)/REPS) / (REPS-1)
      WRITE(16,19) AVEPERM,PCT
      WRITE(16,20) AVEQ,VARQ
C
      STOP
      END
C
C
```

```
C*****************************************************************
C
C  SUBROUTINE BPERM
C
C    PURPOSE
C      TO PERFORM AN APPROXIMATE PERMUTATION TEST BY SAMPLING
C      1000 TIMES FROM THE SET OF ALL POSSIBLE PERMUTATIONS
C
C    USAGE
C      CALL BPERM(ALL,EXTREM,IP)
C
C
C    DESCRIPTON OF PARAMETERS
C       ALL     -  LOGICAL MARKER SIGNIFYING AN ABORTED PERMUTATION
C                LOOP MEANING P-VALUE FOR ALL TESTS GREATER THAN .05
C       EXTREM -  VECTOR COUNTING EXTREM VALUES OF THE STATISTICS
C
C    SUBROUTINES CALLED
C       SAMPER
C
C
      SUBROUTINE BPERM(ALL,EXTREM,IP)
      REAL*6 OSTAT(10),PSTAT(10),COMB(20),CVAL,PSAMP1(10),PSAMP2(10)
      INTEGER*3 IC,JC,N,M,NSTAT,IP,NPERM,INT,ISAM(20),EXTREM(10)
      LOGICAL*3 ALL,CONTIN(10)
      COMMON/PERMCOM/OSTAT,NSTAT,N,M,COMB,INT,NPERM,CVAL
C
      DO 100 K=1,NSTAT
       CONTIN(K) = .TRUE.
       PSTAT(K) = 0.0
  100  EXTREM(K) = 0
C
       IP=0
      DO 200  I=1,NPERM
       IP = IP + 1
C
        CALL SAMPER(ISAM,N,M)
C
        IC=1
        JC=1
         DO 110  L=1,N+M
           IF (ISAM(L) .EQ. 1) THEN
                  PSAMP1(IC) = COMB(L)
                  IC = IC + 1
           ELSE
                  PSAMP2(JC) = COMB(L)
                  JC = JC + 1
           END IF
  110    CONTINUE
C
```

```
C
C    CALCULATE THE STATISTICS
C
C      K=1 : MC(0) -- MIDRANGE
C      K=2 : MC(.2)
C      K=3 : MC(.3)
C      K=4 : MC(.5) = M(0) -- MEAN
C      K=5 : M(.2)
C      K=6 : M(.3)
C      K=7 : M(.5) -- MEDIAN
C      K=8 : ADAPTIVELY CHOSEN TO BE ONE OF THE ABOVE
C
C
        DO 185  K=1,NSTAT
           IF (.NOT. CONTIN(K)) THEN
                  GOTO 185
           ELSE
                  GOTO (120,125,130,140,145,150,160,165),K
           END IF
C
  120         CALL MIDSCL(PSAMP1,PSAMP2,N,M,SCALE1,SCALE2)
              GOTO 170
C
  125         ALPHA=.2
              GOTO 135
C
  130         ALPHA=.3
C
  135         CALL TCMNSC(PSAMP1,PSAMP2,N,M,SCALE1,SCALE2,ALPHA)
              GOTO 170
C
  140         CALL MNSCAL(PSAMP1,PSAMP2,N,M,SCALE1,SCALE2)
              GOTO 170
C
  145         ALPHA=.2
              GOTO 155
C
  150         ALPHA=.3
C
  155         CALL TMNSCL(PSAMP1,PSAMP2,N,M,SCALE1,SCALE2,ALPHA)
              GOTO 170
C
  160         CALL MEDSCL(PSAMP1,PSAMP2,N,M,SCALE1,SCALE2)
              GOTO 170
C
  165         PSTAT(K) = PSTAT(INT)
              GOTO 185
C
  170      PSTAT(K) = RATIO(SCALE1,SCALE2)
  185   CONTINUE
```

```
C
       ALL = .FALSE.
        DO 190  K=1,NSTAT
          IF (.NOT. CONTIN(K)) THEN
                GOTO 190
          ELSE
                IF (PSTAT(K) .GT. OSTAT(K)) EXTREM(K) = EXTREM(K) + 1
                IF (EXTREM(K) .GT. CVAL) CONTIN(K) = .FALSE.
                IF (CONTIN(K)) ALL = .TRUE.
          END IF
  190    CONTINUE
C
C*     WRITE(16,191) ALL
C*191  FORMAT(' THE VALUE OF ALL IS: ',L2)
       IF (.NOT. ALL) GOTO 210
  200 CONTINUE
  210 RETURN
      END
C
C
C*****************************************************************
C
C  SUBROUTINE DEVSQ
C    PURPOSE
C      SUBTRACT A QUANTITY FROM THE SAMPLE VECTOR AND SQUARE
C      THOSE DEVIATIONS
C
C    USAGE
C      CALL DEVSQ(SAMPL1,SAMPL2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1 (2) -  REAL*6 ARRAY OF SIZE N (M) CONTAINING
C                     SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)    -  LOCATION ESTIMATES FOR SAMPLE 1 (2)
C      SQDEV1 (2)  -  THE SQARED DEVIATION FOR SAMPLE 1 (2)
C
      SUBROUTINE DEVSQ(X,Y,N,M,TM1,TM2,Z,W)
      REAL*6 X(N),Y(M),TM1,TM2,Z(N),W(M)
      INTEGER*3 N,M
C
      DO 100  I=1,N
  100  Z(I) = (X(I) - TM1) ** 2
C

      DO 110  I=1,M
  110  W(I) = (Y(I) - TM2) ** 2
C
      RETURN
      END
C
C
```

```
C*****************************************************************
C
C  SUBROUTINE MEAN
C
C    PURPOSE
C      CALCULATES THE SAMPLE MEAN FOR EACH OF TWO SAMPLES
C
C    USACE
C      CALL MEAN(SAMPL1,SAMPL2,N,M,LOC1,LOC2)
C
C    DESCRIPTION OF PARAMETERS
C      SAMPl (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC THE
C                    SAMPLE VALUES
C      LOC1 (2)   -  ESTIMATE OF THE LOCATION PARAMETER (THE MEAN)
C                    FOR SAMPLE 1 (2)
C
C
      SUBROUTINE MEAN(X,Y,N,M,MEAN1,MEAN2)
      REAL*6 X(N),Y(M),SUM1,SUM2,MEAN1,MEAN2
      INTECER*3 N,M
C
      SUM1=0.0
      SUM2=0.0
C
      DO 100  I=1,N
  100 SUM1 = SUM1 + X(I)
      MEAN1 = SUM1 / FLOAT(N)
C
      DO 120  I=1,M
  120 SUM2 = SUM2 + Y(I)
      MEAN2 = SUM2 / FLOAT(M)
C
      RETURN
      END
C
C
C*****************************************************************
C
C  SUBROUTINE MNSCAL
C    PURPOSE
C      CALCULATES AN ESTIMATE OF SCALE BASED ON THE MEAN FOR EACH
C      OF TWO SAMPLES
C
C    USACE
C      CALL MNSCAL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
C
C    SUBROUTINES CALLED
C      MEAN,DEVSQ
```

```
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1 (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC
C                     SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)    -  ESTIMATE OF LOCATION BASED ON SAMPLE 1 (2)
C      SCALE1 (2)  -  RETURNED VALUE OF THE ESTIMATE OF SCALE
C                     FROM SAMPLE 1 (2)
C
C
      SUBROUTINE MNSCAL(SAMP1,SAMP2,N,M,SCALE1,SCALE2)
      REAL*6 SAMP1(10),SAMP2(10),LOC1,LOC2,SCALE1,SCALE2,
     1       SQDEV1(10),SQDEV2(10),SC1,SC2
      INTECER*3 N,M
C
      CALL MEAN(SAMP1,SAMP2,N,M,LOC1,LOC2)
C
      CALL DEVSQ(SAMP1,SAMP2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
      CALL MEAN(SQDEV1,SQDEV2,N,M,SC1,SC2)
C
      SCALE1=SC1
      SCALE2=SC2
C
      RETURN
      END
C
C
C******************************************************************
C
C  SUBROUTINE MEDIAN
C
C    PURPOSE
C      CALCULATES THE SAMPLE MEDIAN FOR EACH OF TWO SAMPLES
C
C    USACE
C      CALL MEDIAN(SAMPL1,SAMPL2,N,M,LOC1,LOC2)
C
C    SUBROUTINES CALLED
C      SHELL
C
C    DESCRIPTION OF PARAMETERS
C      SAMP1 (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC THE
C                    SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)   -  ESTIMATE OF THE LOCATION PARAMETER (THE MEDI)
C                    FOR EACH SAMPLE
C
C
```

```
      SUBROUTINE MEDIAN(X,Y,N,M,MEDI1,MEDI2)
      REAL*6 X(N),Y(M),MEDI1,MEDI2
      INTEGER*3 N,M
      LOGICAL ODD
C
      MEDI1=0.0
      MEDI2=0.0
C
      CALL SHELL(X,N)
C
      ODD=.FALSE.
      IF (MOD(N,2) .NE. 0.0)ODD=.TRUE.
C
      IF (ODD) THEN
                     MEDI1 = X( (N+1)/2 )
      ELSE
                     MEDI1 = ( X( N/2 )  +  X( N/2 + 1) ) / 2.
      ENDIF
C
C
      CALL SHELL(Y,M)
C
      ODD=.FALSE.
      IF (MOD(M,2) .NE. 0.0)ODD=.TRUE.
C
      IF (ODD) THEN
                     MEDI2 = Y( (M+1)/2 )
      ELSE
                     MEDI2 = ( Y( M/2 )  +  Y( M/2 + 1) ) / 2.
      END IF
C
      RETURN
      END
C
C
C*******************************************************************
C
C  SUBROUTINE MEDSCL
C    PURPOSE
C      CALCULATES AN ESTIMATE OF SCALE BASED ON THE MEDIAN FOR EACH
C      OF TWO SAMPLES
C
C    USAGE
C      CALL MEDSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
C
C    SUBROUTINES CALLED
C      MEDIAN,DEVSQ
C
```

```
C     DESCRIPTION OF PARAMETERS
C       SAMPL1 (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC
C                      SAMPLE VALUES FROM POPULATION 1 (2)
C       LOC1 (2)    -  ESTIMATE OF LOCATION BASED ON SAMPLE 1 (2)
C       SCALE1 (2   -  RETURNED VALUE OF THE ESTIMATE OF SCALE
C                      FROM SAMPLE 1 (2)
C
C
      SUBROUTINE MEDSCL(SAMP1,SAMP2,N,M,SCALE1,SCALE2)
      REAL*6 SAMP1(10),SAMP2(10),LOC1,LOC2,SCALE1,SCALE2,
     1       SQDEV1(10),SQDEV2(10),SC1,SC2
      INTECER*3 N,M
C
      CALL MEDIAN(SAMP1,SAMP2,N,M,LOC1,LOC2)
C
      CALL DEVSQ(SAMP1,SAMP2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
      CALL MEDIAN(SQDEV1,SQDEV2,N,M,SC1,SC2)
C
      SCALE1=SC1
      SCALE2=SC2
C
      RETURN
      END
C
C
C*****************************************************************
C  SUBROUTINE MIDRAN
C
C     PURPOSE
C       CALCULATES THE MIDRANCE FOR EACH OF TWO SAMPLES
C
C     USACE
C       CALL MIDRAN(SAMPL1,SAMPL2,N,M,LOC1,LOC2)
C
C
C     SUBROUTINES CALLED
C       SHELL
C
C     DESCRIPTION OF PARAMETERS
C       SAMPL (2)   -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC THE
C                      SAMPLE VALUES
C       LOC1 (2)    -  ESTIMATE OF THE LOCATION PARAMETER (MIDRANGE)
C                      FOR EACH SAMPLE
C
C
```

```
      SUBROUTINE MIDRAN(X,Y,N,M,MIDRA1,MIDRA2)
      REAL*6 X(N),Y(M),MIDRA1,MIDRA2
      INTECER*3 N,M
C
      MIDRA1 = 0.0
      MIDRA2 = 0.0
C
      CALL SHELL(X,N)
C
      MIDRA1 = ( X(1) + X(N) ) / 2.0
C
      CALL SHELL(Y,M)
C
      MIDRA2 = ( Y(1) + Y(M) ) / 2.0
C
      RETURN
      END
C
C
C*************************************************************
C
C  SUBROUTINE MIDSCL
C
C    PURPOSE
C      CALCULATES AN ESTIMATE OF SCALE BASED ON THE MIDRANCE FOR
C      EACH OF TWO SAMPLES
C
C    USACE
C      CALL MIDSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2)
C
C    SUBROUTINES CALLED
C      MIDRAN,DEVSQ
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1 (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC
C                     SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)    -  ESTIMATE OF LOCATION BASED ON SAMPLE 1 (2)
C      SCALE1 (2)  -  RETURNED VALUE OF THE ESTIMATE OF SCALE
C                     FROM SAMPLE 1 (2)
C
C
      SUBROUTINE MIDSCL(SAMP1,SAMP2,N,M,SCALE1,SCALE2)
      REAL*6 SAMP1(10),SAMP2(10),LOC1,LOC2,SCALE1,SCALE2,
     1       SQDEV1(10),SQDEV2(10),SC1,SC2
      INTECER*3 N,M
C
      CALL MIDRAN(SAMP1,SAMP2,N,M,LOC1,LOC2)
C
      CALL DEVSQ(SAMP1,SAMP2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
      CALL MIDRAN(SQDEV1,SQDEV2,N,M,SC1,SC2)
C
```

```
      SCALE1=SC1
      SCALE2=SC2
C
      RETURN
      END
C
C
C*****************************************************************
C
C  FUNCTION QHAT
C
C    PURPOSE
C      CALCULATES Q, THE NONNORMALITY INDICATOR BY WHICH ALPHA IS
C      DETERMINED ADAPTIVELY (SEE HOGG, 1974)
C
C    USAGE
C      QVAL = QHAT(SAMP,N)
C
C    SUBROUTINES CALLED
C      SHELL
C
C    DESCRIPTION OF PARAMETERS
C      SAMP  -  REAL*6 ARRAY OF SIZE N CONTAINING SAMPLE VALUES
C               FROM A POPULATION
C
C
      FUNCTION QHAT(X,N)
      REAL*6 X(N),HOLD1,HOLD2,QHAT
      INTEGER*3 N,INUM,IDEN
C
      CALL SHELL(X,N)
C
      INUM = 0.05*N
      IDEN = 0.5*N
      HOLD1 = 0.0
      HOLD2 = 0.0
C
      IF (INUM .LT. 1) GOTO 110
      DO 100  I=1,INUM
  100 HOLD1 = HOLD1 + X(N+1-I) - X(I)
  110 HOLD1 = HOLD1 + (.05*N - INUM) * ( X(N-INUM) - X(INUM+1) )
      HOLD1 = HOLD1/(.05*N)
C
      DO 120  I=1,IDEN
  120 HOLD2 = HOLD2 + X(N+1-I) - X(I)
      HOLD2 = HOLD2/(.5*N)
      IF (HOLD2 .LT. 0.000001) HOLD2=0.000001
C
      QHAT = HOLD1/HOLD2
      RETURN
      END
C
```

```
C*******************************************************************
C
C  SUBROUTINE QINT
C
C    PURPOSE
C      DETERMINES THE INTERVAL IN WHICH Q IS OBSERVED IN ORDER
C      TO CHOOSE THE BEST TRIMMED MEAN AS SUGGESTED BY PRESCOTT
C      (SEE BOYER AND KOLSON, 1983)
C
C    USACE
C      CALL QINT(SAMPL1,SAMPL2,N,M,Q,INT)
C
C    FUNCTIONS USED
C      QHAT
C
C    DESCRIPTION OF PARAMETERS
C      SAMPl (2) -  REAL*6 ARRAY OF SIZE N CONTAININC SAMPLE VALUES
C      Q         -  THE ESTIMATED VALUE OF HOGG'S Q STATISTIC
C      INT          THE INTERVAL (2,6) WHEREIN QHAT LIES
C
C
      SUBROUTINE QINT(X,Y,N,M,Q,INT)
      REAL*6 X(N),Y(M),Q,QVAL1,QVAL2
      INTEGER*3 N,M,INT
C
      QVAL1 = QHAT(X,N)
      QVAL2 = QHAT(Y,M)
      Q = (QVAL1 + QVAL2) / 2.0
C
      IF ( Q .LT. 2.2 ) THEN
                              INT = 2
      ELSE
           IF ( Q .LT. 2.4) THEN
                                   INT = 3
           ELSE
                IF ( Q .LE. 2.8) THEN
                                        INT = 4
                ELSE
                     IF ( Q .LE. 3.0) THEN
                                             INT = 5
                     ELSE
                          INT = 6
                     END IF
                END IF
           END IF
      END IF
C
C*    WRITE(16,100)Q,INT
C*100 FORMAT(' THE VALUE OF Q IS ',F7.5,' PLACED IN INTERVAL ',I2)
C
     RETURN
     END
```

```
C*****************************************************************
C
C  FUNCTION RATIO
C
C    PURPOSE
C      CALCULATE THE RATIO OF TWO STATISTICS
C
C    USACE
C      STAT = RATIO(SCALE1,SCALE2)
C
C    DESCRIPTION OF PARAMETERS
C      SCALE1  -  SCALE ESTIMATE OF A SAMPLE FROM A POPULATION
C                 HAVINC SMALLER ACTUAL SCALE
C
C      SCALE2  -  SCALE ESTIMATE OF A SAMPLE FROM A POPULATION
C                 HAVINC LARCER ACTUAL SCALE
C
C
      FUNCTION RATIO(SC1,SC2)
      REAL*6 SC1,SC2,RATIO
C
        IF (SC1 .LT. 0.00001) SC1 = 0.00001
        RATIO = SQRT(SC2) / SQRT(SC1)
C
      RETURN
      END
C
C
C*****************************************************************
C
C  SUBROUTINE SAMPER
C
C    PURPOSE
C      SAMPLE AN ELEMENT RANDOMLY FROM THE SET OF ALL POSSIBLE
C      PERMUTATIONS
C
C    USACE
C      CALL SAMPER(ISAM,N,M)
C
C    FUNCTION CALLED
C      RANU
C
C    DESCRIPTION OF PARAMETERS
C      ISAM  -  RETURNED INDICATOR ARRAY OF LENCTH N+M
C      N,M   -  SAMPLE SIZES
C
C    METHOD
C      THE ARRAY ISAM IS USED TO INDICATE THE ELEMENTS OF THE
C      COMBINED SAMPLE THAT WILL BE ASSICNED TO SAMPLE 1
C      (INDICATOR=1) OR SAMPLE 2 (INDICATOR=0) FOR THE RANDOMLY
C      SELECTED PERMUTATION.  THE ELEMENTS OF ISAM ARE INITIALIZED
C      TO 0 AND TURNED TO 1 BY RANDOM SAMPLINC WITHOUT REPLACEMENT.
```

```
C
      SUBROUTINE SAMPER(ISAM,N,M)
      INTECER*3 N,M,I,NSAM,ISAM(N+M)
      REAL*6 U,C
C
      DO 100  L=1,N+M
  100 ISAM(L)=0
      C=FLOAT(N+M)
      NSAM=0
C
  150 U = RANU(0.0,1.0)
      I=INT(U*C) + 1
      IF (ISAM(I) .EQ. 1) GOTO 150
      ISAM(I)=1
      NSAM=NSAM+1
      IF (NSAM .LT. N) COTO 150
C
      RETURN
      END
C
C
C*****************************************************************
C
C  SUBROUTINE SHELL
C
C    PURPOSE
C      SORT A SET OF DATA INTO ASCENDING ORDER
C
C    USAGE
C      CALL SHELL(SAMP,NSIZE)
C
C    DESCRIPTION OF PARAMETERS
C      SAMP   -  ARRAY OF SAMPLE DATA TO BE SORTED
C      NSIZE  -  SIZE OF SAMPLE
C
C    METHOD
C      SHELL SORT TECHNIQUE
C
      SUBROUTINE SHELL(SAMP,NSIZE)
      REAL*6 SAMP(NSIZE),T
      INTEGER*3 S,NSIZE
C
      S=NSIZE
  100 S=INT(S/2)
      IF (S .LT. 1)GOTO 150
```

```
      DO 140  K=1,S
       DO 130  I=K,NSIZE-S,S
        J=I
        T=SAMP(I+S)
  110   IF (T .GE. SAMP(J)) GOTO 120
          SAMP(J+S)=SAMP(J)
          J=J-S
          IF (J .GE. 1) GOTO 110
  120   SAMP(J+S)=T
  130  CONTINUE
  140 CONTINUE
      GOTO 100
C
  150 RETURN
      END
C
C
C*****************************************************************
C
C  FUNCTION TCMEAN
C
C    PURPOSE
C      CALCULATES THE MEAN OF THE TRIMMINGS DEFINED BY ALPHA
C
C    USAGE
C      STAT = TCMEAN(SAMP,N,ALPHA)
C
C    DESCRIPTION OF PARAMETERS
C      SAMP   -  REAL*6 ARRAY OF SIZE N CONTAINING THE SAMPLE
C                VALUES FROM A POPULATION
C      ALPHA  -  THE PERCENT OF TRIMMING DESIRED
C
C
      FUNCTION TCMEAN(X,N,A)
      REAL*6 X(N),A,TCSUM,TCMEAN,DIV
      INTEGER*3 N,I1,I2,ISTART
C
      CALL SHELL(X,N)
C
      IF (A .LT. .00001)A=.00001
      DIV = 2. * N * A
      ISTART = N * A
      TCSUM = 0.0
      IF (ISTART .LT. 1) GOTO 110
      DO 100  I=1,ISTART
  100 TCSUM = TCSUM + X(N+1-I) + X(I)
  110 TCSUM = TCSUM + (N*A - ISTART) * ( X(ISTART+1) + X(N-ISTART) )
      TCMEAN = TCSUM / DIV
C
      RETURN
      END
C
```

```
C
C*****************************************************************
C
C  SUBROUTINE TCMNSC
C
C    PURPOSE
C      CALCULATES AN ESTIMATE OF SCALE BASED ON THE DESIGNATED
C      MEAN OF TRIMMINGS FOR EACH OF TWO SAMPLES
C
C    USACE
C      CALL TCMNSC(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2,ALPHA)
C
C    SUBROUTINES/FUNCTIONS CALLED
C      TCMEAN,DEVSQ
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1 (2)  -  REAL*6 ARRAY OF LENCTH N (M) CONTAININC
C                     SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)    -  ESTIMATE OF LOCATION BASED ON SAMPLE 1 (2)
C      SCALE1 (2)  -  RETURNED VALUE OF THE ESTIMATE OF SCALE
C                     FROM SAMPLE 1 (2)
C      ALPHA       -  THE AMOUNT OF TRIMMING REQUESTED
C
C
      SUBROUTINE TCMNSC(SAMP1,SAMP2,N,M,SCALE1,SCALE2,A)
      REAL*6 SAMP1(10),SAMP2(10),LOC1,LOC2,SCALE1,SCALE2,
     1       SQDEV1(10),SQDEV2(10),A
      INTEGER*3 N,M
C
      LOC1 = TCMEAN(SAMP1,N,A)
      LOC2 = TCMEAN(SAMP2,M,A)
C
      CALL DEVSQ(SAMP1,SAMP2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
      SCALE1 = TCMEAN(SQDEV1,N,A)
      SCALE2 = TCMEAN(SQDEV2,M,A)
C
      RETURN
      END
C
C
C*****************************************************************
C
C  FUNTION TMEAN
C
C    PURPOSE
C      CALCULATES THE ALPHA TRIMMED MEAN FROM A SAMPLE
C
C    USACE
C      LOC = TMEAN(SAMP,N,ALPHA)
C
```

```
C     DESCRIPTION OF PARAMETERS
C       SAMP   -  REAL*6 ARRAY OF SIZE N CONTAINING THE SAMPLE
C                 VALUES FROM A POPULATION
C       ALPHA  -  THE PERCENT OF TRIMMING DESIRED
C
C
      FUNCTION TMEAN(X,N,A)
      REAL*6 X(N),A,TSUM,TMEAN,DIV
      INTEGER*3 N,I1,I2,ISTART
C
       CALL SHELL(X,N)
C
      IF ( A .GT. .499999)A=.499999
      DIV = N - 2.0*N*A
      ISTART = N*A
      I1 = ISTART + 2
      I2 = N - ISTART - 1
      TSUM = 0.0
      IF (I1 .GT. I2) GOTO 110
      DO 100  I = I1,I2
  100 TSUM = TSUM + X(I)
  110 TSUM = TSUM + (1.0 + ISTART - N*A ) * ( X(ISTART+1) + X(I2+1) )
      TMEAN = TSUM / DIV
C
      RETURN
      END
C
C
C*****************************************************************
C
C  SUBROUTINE TMNSCL
C    PURPOSE
C      CALCULATES AN ESTIMATE OF SCALE BASED ON THE DESIGNATED
C      TRIMMED MEAN FOR EACH OF TWO SAMPLES
C
C    USAGE
C      CALL TMNSCL(SAMPL1,SAMPL2,N,M,SCALE1,SCALE2,ALPHA)
C
C    SUBROUTINES/FUNCTIONS CALLED
C      TMEAN,DEVSQ
C
C    DESCRIPTION OF PARAMETERS
C      SAMPL1 (2)  -  REAL*6 ARRAY OF LENGTH N (M) CONTAINING
C                     SAMPLE VALUES FROM POPULATION 1 (2)
C      LOC1 (2)    -  ESTIMATE OF LOCATION BASED ON SAMPLE 1 (2)
C      SCALE1 (2)  -  RETURNED VALUE OF THE ESTIMATE OF SCALE
C                     FROM SAMPLE 1 (2)
C      ALPHA       -  THE AMOUNT OF TRIMMING REQUESTED
C
C
```

```
      SUBROUTINE TMNSCL(SAMP1,SAMP2,N,M,SCALE1,SCALE2,A)
      REAL*6 SAMP1(10),SAMP2(10),LOC1,LOC2,SCALE1,SCALE2,
     1       SQDEV1(10),SQDEV2(10),A
      INTEGER*3 N,M
C
      LOC1 = TMEAN(SAMP1,N,A)
      LOC2 = TMEAN(SAMP2,M,A)
C
      CALL DEVSQ(SAMP1,SAMP2,N,M,LOC1,LOC2,SQDEV1,SQDEV2)
C
      SCALE1 = TMEAN(SQDEV1,N,A)
      SCALE2 = TMEAN(SQDEV2,M,A)
C
      RETURN
      END
C
C
C*******************************************************************
```

Appendix 2

Listing of Simulation Results

Power Tables and Figures

Legend

Test Statistic	Plot Character
$m^c(0.0)$	diamond
$m^c(0.5)$	square
$m\ (0.5)$	triangle
adaptive	star

TABLE A-1

Simulation Results
.05 Rejection Rates

Uniform Distribution

	$\theta=1$	$\theta=1.5$	$\theta=2$	$\theta=3$
$m^c(0.0)$	0.048	0.501	0.824	0.973
$m^c(0.2)$	0.048	0.494	0.814	0.973
$m^c(0.3)$	0.055	0.489	0.799	0.969
$m^c(0.5)$	0.047	0.436	0.756	0.963
m (0.2)	0.050	0.295	0.538	0.818
m (0.3)	0.045	0.243	0.452	0.736
m (0.5)	0.046	0.205	0.395	0.630
adaptive	0.048	0.494	0.816	0.974

TABLE A-2

Simulation Results
.05 Rejection Rates

Prescott(.25) Distribution

	$\theta=1$	$\theta=2$	$\theta=3$	$\theta=4$
$m^c(0.0)$	0.052	0.663	0.933	0.981
$m^c(0.2)$	0.054	0.658	0.933	0.981
$m^c(0.3)$	0.052	0.674	0.936	0.980
$m^c(0.5)$	0.050	0.682	0.931	0.975
m (0.2)	0.051	0.494	0.817	0.911
m (0.3)	0.045	0.436	0.731	0.867
m (0.5)	0.047	0.359	0.631	0.780
adaptive	0.056	0.669	0.936	0.979

TABLE A-3

Simulation Results
.05 Rejection Rates

Normal Distribution

	$\theta=1$	$\theta=2$	$\theta=3$	$\theta=4$
$m^c(0.0)$	0.045	0.533	0.833	0.940
$m^c(0.2)$	0.047	0.530	0.829	0.941
$m^c(0.3)$	0.042	0.544	0.854	0.956
$m^c(0.5)$	0.047	0.569	0.871	0.958
m (0.2)	0.049	0.463	0.758	0.889
m (0.3)	0.052	0.410	0.699	0.835
m (0.5)	0.045	0.351	0.602	0.764
adaptive	0.051	0.546	0.857	0.955

TABLE A-4

Simulation Results
.05 Rejection Rates

Prescott(.75) Distribution

	$\theta=1$	$\theta=2$	$\theta=3$	$\theta=4$
$m^c(0.0)$	0.052	0.399	0.697	0.877
$m^c(0.2)$	0.049	0.397	0.696	0.869
$m^c(0.3)$	0.049	0.430	0.715	0.895
$m^c(0.5)$	0.054	0.450	0.739	0.907
m (0.2)	0.060	0.410	0.658	0.851
m (0.3)	0.056	0.377	0.597	0.806
m (0.5)	0.048	0.327	0.546	0.733
adaptive	0.055	0.444	0.737	0.908

TABLE A-5

Simulation Results
.05 Rejection Rates

Double Exponential Distribution

	$\theta=1$	$\theta=2$	$\theta=4$	$\theta=6$
$m^c(0.0)$	0.044	0.326	0.793	0.911
$m^c(0.2)$	0.045	0.320	0.784	0.914
$m^c(0.3)$	0.046	0.342	0.807	0.928
$m^c(0.5)$	0.048	0.359	0.826	0.945
m (0.2)	0.057	0.327	0.780	0.924
m (0.3)	0.054	0.295	0.750	0.900
m (0.5)	0.052	0.271	0.694	0.847
adaptive	0.059	0.365	0.823	0.948

TABLE A-6

Simulation Results
.05 Rejection Rates

Mixed Normal Distribution

	θ=1	θ=2	θ=4	θ=6
$m^c(0.0)$	0.048	0.297	0.555	0.706
$m^c(0.2)$	0.047	0.295	0.552	0.716
$m^c(0.3)$	0.047	0.302	0.572	0.746
$m^c(0.5)$	0.047	0.320	0.595	0.763
m (0.2)	0.042	0.336	0.778	0.909
m (0.3)	0.046	0.313	0.749	0.881
m (0.5)	0.056	0.287	0.683	0.836
adaptive	0.055	0.364	0.719	0.861

Note: Population 1 was N(0,1) contaminated with 10% N(0,64). If, for example, $\theta_2/\theta_1 = 3$, then Population 2 was N(0,9) contaminated with 10% N(0,576).

TABLE A-7

Simulation Results
.05 Rejection Rates

Cauchy Distribution

	$\theta=1$	$\theta=3$	$\theta=5$	$\theta=8$
$m^c(0.0)$	0.052	0.318	0.478	0.618
$m^c(0.2)$	0.049	0.315	0.479	0.621
$m^c(0.3)$	0.049	0.333	0.498	0.642
$m^c(0.5)$	0.048	0.350	0.520	0.658
$m\ (0.2)$	0.038	0.446	0.687	0.846
$m\ (0.3)$	0.039	0.455	0.693	0.849
$m\ (0.5)$	0.037	0.417	0.674	0.834
adaptive	0.055	0.447	0.691	0.850

FIGURE B-1: UNIFORM DISTRIBUTION

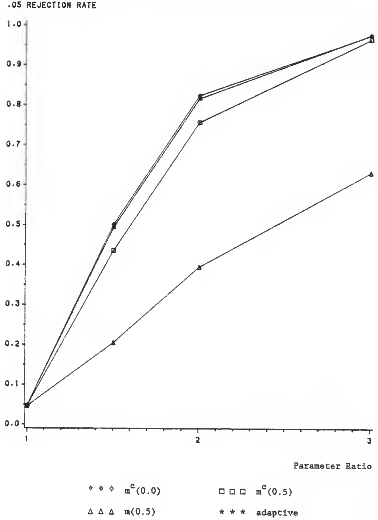


FIGURE B-2: PRESCOTT(.25) DISTRIBUTION

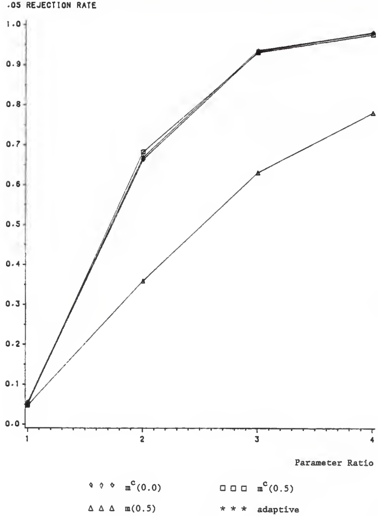


FIGURE B-3: NORMAL DISTRIBUTION

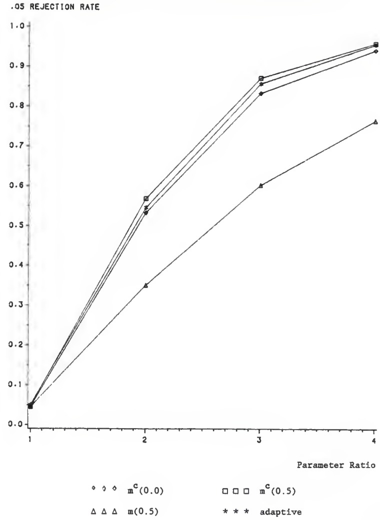


FIGURE B–4: PRESCOTT(.75) DISTRIBUTION

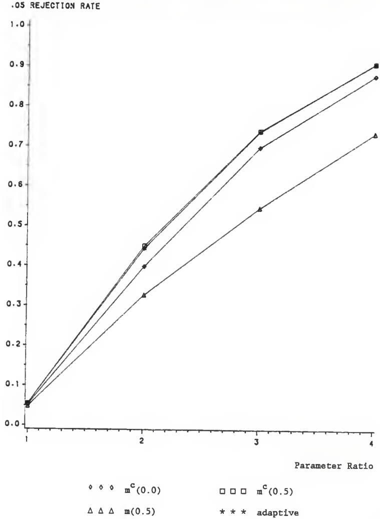


FIGURE B-5: DOUBLE EXPONENTIAL DISTRIBUTION

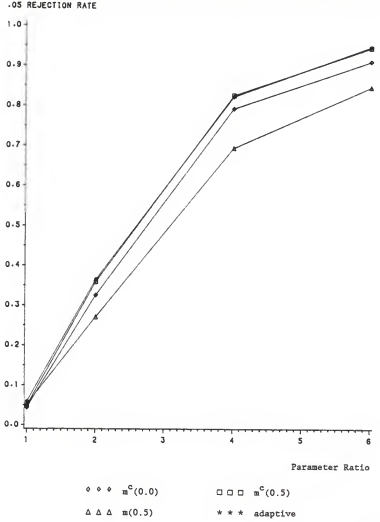


FIGURE B-6: MIXED NORMAL DISTRIBUTION

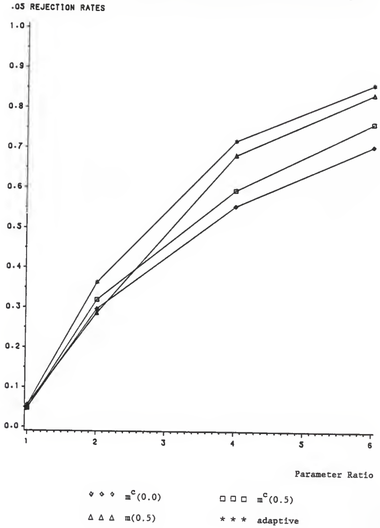
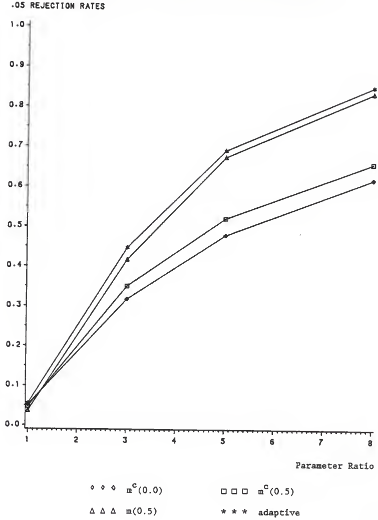


FIGURE B–7: CAUCHY DISTRIBUTION



ADAPTIVE TESTING IN THE TWO SAMPLE SCALE PROBLEM

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ABSTRACT

When testing for equality of scale in two populations, the usual F test has been shown to have undesirable properties when the populations in question are heavy tailed. The test is low in power, but even worse, the demonstrated power of the F test cannot be trusted since it fails to retain the .05 level when testing at the null hypothesis. This report details a study of alternative tests for this two sample scale problem. Specifically chosen for study were seven symmetric populations which vary in tailweight. The power of eight test statistics based on functions of trimmed means (the average of a specified portion of the sample) are compared via permutation tests. Of special interest is an adaptive test procedure, which first estimates the tailweight of the population, then, based on that estimate, chooses the amount of trimming used in the test statistic. This procedure is shown to be the most consistently powerful of the tests studied here.